The History of the Application of Incomplete Integration for the Control of Laser Systems

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Abstract: The present paper explores the possibility of achieving a commensurate or better result by a simpler controller in comparison with the well-known sophisticated methods of synthesizing $PI^{\lambda}D^{\mu}$-regulators. It is shown that these regulators are more complex than the well-known and successfully used PID-regulators. The possibility of achieving better results in an easier manner is demonstrated. The requirements for an ideal linear automatic control system in the form of an ideal amplitude-frequency characteristic are justified. The methods of designing such regulators are given and examples showing their advantage are presented.

Key words: PID-regulator, feedback, control, automatics, dynamic error, static error, numerical optimization

INTRODUCTION

In this paper, we discuss some specific issues in the design of regulators for automatic systems. The presentation is based on modeling results in software *VisSim* [17], which has several versions. In the case where it does not matter, the version number, we will use the name of this software without specifying this number; in other cases, the version number is indicated at the end of the name, for example, *VisSim* 5.0.

The software complex *Simulink* + *MATLAB* (along with the set of packages for their expansion *Toolbox* and *Blockset*), as *V. Dyakonov* rightly notes [17], proved too cumbersome for such relatively simple applications as modeling and optimization of regulators. This package takes up a noticeably large amount of memory, an excessively large library of blocks, most of which are unnecessarily specialized and are not required in most of the tasks to be solved; the created files also occupy a fairly large amount of memory, in total this amounts to several gigabytes. For this reason, there has recently been a sharp increase in interest in a small-scale but sufficient universal system of block imitation visual-oriented mathematical modeling, such as *VisSim*. This software has been created by the corporation *Visual Solution Inc*. (*USA*). The main developer of the software and the Head of the corporation is *Peter Darnell*. Along with the system itself, a number of packages for its expansion have been released, significantly increasing the already tangible capabilities of this system.

This "pearl" in the world of mathematical modeling programs has long attracted the interest of specialists in the field of mathematical modeling. For example, the corporation *MathSoft*, the creator of the famous and most popular computer mathematics *Mathcad*, not only ensured the docking of this system with the *VisSim*, but also began to supply *VisSim* with some versions of the *Mathcad* package [17]. The *VisSim* version can also be integrated with the *Simulink* + *MATLAB* system [17].

A major contribution to the distribution of the *VisSim* was made by the site www.vissim.nm.ru, created under the direction of *N. Klimachev* from the South Ural University, some versions of this program, for example, *VisSim* 3.0, are distributed free of charge, later versions can be used within 60 days for free, these versions can be obtained from the website www.vissim.com [17].

I. FRACTIONAL-POWER PID-REGULATORS

The control of dynamic objects in a locked loop is applied in all branches of science and technology, with the most common regulators containing a proportional, integrating and derivative link, called PID-regulators. Many authors attempted to modify the structure of PID-regulators in terms of novelty, based on the best efficiency. One of these directions is to use a non-integer exponent in the differential equation of the regulator, which corresponds to incomplete integration and (or) differentiation [1–15].

The essence of the so-called regulator with a fractional order of integration and (or) derivation ($PI^{\lambda}D^{\mu}$-regulator, fractional-power regulators) consists in two steps: a) it is asserted and shown by examples that the use of a non-integral degree of integration and (or) derivation gives a positive effect (at least in a number of tasks); b) it is asserted that within the framework of a preassigned accuracy, the regulator of the previous point can be implemented in a structure using only a whole degree of integration and derivation. Recommendations for such an approximation are given.

Such a decision makes the solution of a number of issues relevant.

1. The more complex regulator, the more complex approximation. Therefore, a question arises – what is the limit in accuracy error or in other words, what is the required complexity of a regulator?

2. How valid are the conclusions from the comparison of the efficiency of complex regulator structures with the efficiency of PID-regulators? It may turn out that the numerical optimization of the parameters of these complex structures provides better regulators than a method based on the approximate implementation of fractional-power regulators.

3. Is it possible to simplify these complex structures based not on the criterion of a small approximation error but based on the criterion for achieving the goals of the regulator' design?

The regulators proposed up to now contain complex structures and the number of numerical parameters in them varies from 6 to 12 or even more. In comparison, PID-regulators contain only three numerical parameters. If the introduction of additional structures with new parameters lead to the better quality control, then there are reasons to assume that a sequential increasing in the number of PID-regulator parameters from 3 to 6 (in some structures) may gradually increase the quality of the regulator and expand its capabilities when moving from a three-parameter regulator to a regulator, for example, with ten parameters.

2. FORMATION OF THE PI$^{\lambda}$(D$^{\mu}$)-REGULATOR

The PI$^{\lambda}$D$^{\mu}$-regulator is proposed to be designed in the form of approximation by a rational transfer function [3–4], viz.

$$C(s) = K_P\left(1+\frac{1}{T_I s^{\lambda}}+T_D s^{\mu}\right), \qquad (1)$$

where K_P is the coefficient of proportional tract, T_I is the parameter of integration link, T_D is the coefficient of derivation link, λ is the order of integrator, such that $0<\lambda<1$, μ is the order of the differentiator, such that s is the argument in the Laplace transform or the symbolic record of the derivation operation in the case of using differential equations. The parameters of the regulator K_P, T_I and T_D in [5] are proposed to be determined using the Ziegel-Nichols method [4]. This choice is clearly not optimal, since this method is not the most effective for achieving the best quality control.

The integration path of the PI$^{\lambda}$D$^{\mu}$-regulator is described in the frequency region by the following transfer function:

$$C_I(s) = \frac{1}{s^{\lambda}}, \qquad (2)$$

In the frequency range [ω_L, ω_H], the fractional-power integrator can be modeled by a function as follows:

$$C_I(s) = \frac{K_I}{\left(1+\dfrac{s}{\omega_C}\right)^{\lambda}}, \qquad (3)$$

at large frequencies the unit in the denominator of the equation (3) can be neglected and we have:

$$C_I(s) = \frac{K_I}{\left(\dfrac{s}{\omega_C}\right)^{\lambda}} = \frac{1}{s^{\lambda}}, \qquad (4)$$

$K_I = (1/\omega_c)^{\lambda}$ and ω_c is the cutoff angular frequency, therefore $\omega_c = 0.1\omega L$.

It is further assumed in [5] that the fractional-power integrator (4) is approximated in the frequency range [ω_L, ω_H] by a rational function in the following form:

$$C_I(s) = \frac{1}{s^{\lambda}} \approx K_I \frac{\prod_{i=0}^{N-1}(1+s/z_i)}{\prod_{i=0}^{N}(1+s/p_i)}. \qquad (5)$$

A method for calculation of the approximation parameters is proposed in [5].

The technique described above for the application of an operational amplifier with a multi-link RC-chain in the feedback is used in the system for stabilizing the frequency of lasers by the resonance of unsaturated absorption. This scheme allows the realization of an incomplete integrating amplifier on one operational amplifier [16]. In this chain, as the sequence number of the resistor and capacitor increase, its value increases a certain number of times. The transfer function $W(\omega)$ of such a link is a product of functions of the form:

$$W(\omega) = \prod_{i=1}^{n} W_i = \prod_{i=1}^{n} \frac{T_{2i+1}p+1}{T_{2i}p+1}.$$

Here $T_{i-1} < T_i < T_{i+1}$.

Fig. 1 shows the scheme of an incomplete integrating amplifier with an average slope of – 10 *dB / dec*. Such an amplifier is also called a semi-integrator. If instead of a resistor at the input we use a capacitor, we get an amplifier that realizes incomplete derivation.

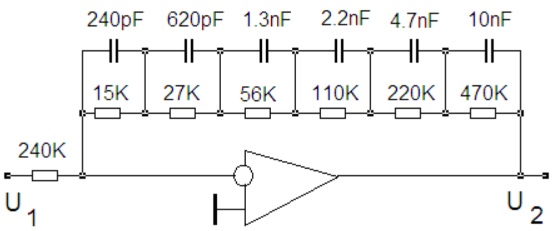

Fig. 1. An amplifier realizing incomplete integration with an average slope of –10 *dB / dec*

With a sufficiently large number of such links, integration alternates with derivation and is blurred in frequency, which gives the average almost constant slope of the logarithmic amplitude-frequency characteristic (LAFC), which is not a multiple of –20 *dB / dec*. Such an integrator in the region of a non-multiple slope has an almost constant phase shift over a sufficiently extended (several decades) section. *Fig*. 2 shows the experimental dependences of the phase characteristics of such an amplifier in various ratios of the parameters of passive elements. Depending on the ratios of the progression of active and reactive elements ($K_1 = R_i / R_{i+1}$ and $K_2 = C_i / C_{i+1}$), an arbitrary value of the slope can be realized in the range from 0 to –20 *dB / dec* and the associated magnitude of the phase delay.

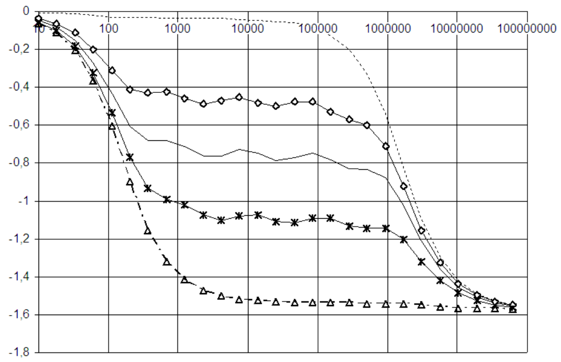

Fig. 2. Phase characteristics of the incomplete integrator for various ratios T_2 / T_1 (in order from top to bottom): –9.5; –5; –3,3; –2; –1.05 (T_i / T_{i-1} = 10, i = 2, ... 9)

The use of incomplete integration in systems with distributed parameters (for example, in systems of thermal stabilization) makes it possible to compensate for undesirable non-standard characteristics of the object. The object in a system of thermal stabilization, as a rule, is not stationary and is characterized by a non-multiple slope generated by the spreading of heat in the physical volume. To optimize the dynamic properties of the system, it may be advisable to perform incomplete integration, which on the one hand, filters out the high-frequency part, but, on the other hand, does not bring the phase shift to a critical value in the vicinity of a single AFC value.

Since the method makes it possible to synthesize LAFC with an arbitrary value of the phase shift, the distributed LAFC parameters of the source object are not a problem, because the phase response can be approximated to an arbitrary desired form in the required region.

The method of obtaining a flat phase-response section with non-stationary gain gives the following advantages:

1. If the phase characteristic over an extended section is constant, then the change in the coefficient affects only the speed of the locked system, but does not change the stability margin. The quality of the transient process remains invariant to the gain.
2. If the phase margin is adjusted to the value corresponding to the maximum permissible overshoot, and the gain is changed, the system becomes optimal in response to the specified criterion.
3. If the phase characteristic of the original object has some form, then it can be transformed into the desired one by adding incomplete integration or derivation.

Fig. 3 shows the experimentally obtained amplitude-frequency and phase-frequency characteristics of the amplifier from *Fig*. 1, depending on the ratio of the time constants. *Fig*. 4 shows the experimental dependence of the phase delay of such an amplifier on the ratio of the time constants.

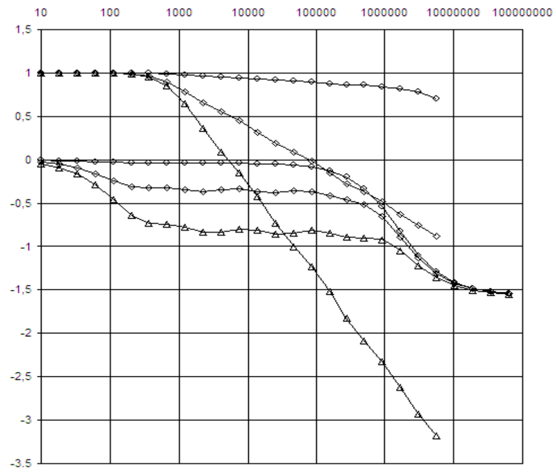

Fig. 3. LAFC and LPFC for T_2 / T_1 = 9.5; 6 and 3.3

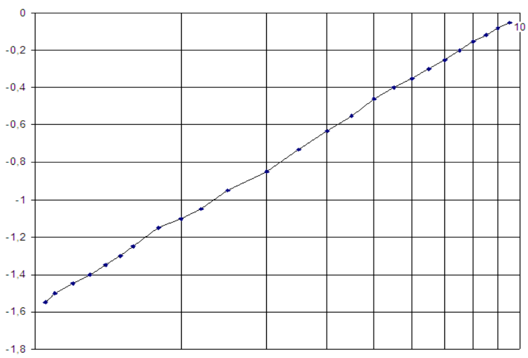

Fig. 4. Dependence of the phase delay of the link with incomplete integration on the ratio of the integration time constants T_{2i} and the differentiation T_{2i+1}

This technique is used in systems for stabilizing the frequency and phase of lasers with the following effect. The slope of the logarithmic amplitude-frequency characteristic of the open control loop in a sufficiently large interval is – 30 dB/dec. This is in absolute magnitude greater than a single slope but less than a double slope. Therefore, the increase in the loop gain in the direction of low frequencies occurs more sharply than in the first-order system, but less sharply than in the second-order system. This makes it possible to more effectively suppress interference in the mid and low frequency range than is achieved in first order systems. The system has a greater stability margin than a second-order system. Thus, the advantages of a system of the first and second order are combined [16].

CONCLUSION

Simulation and numerical optimization has demonstrated that simpler regulators can be designed by the method of numerical optimization with software *VisSim*. It provides better accuracy and better transient processes (and therefore better dynamic accuracy) than the regulators proposed in paper [4]. As a result, both the simplification of the regulator and its improvement have been achieved.

REFERENCES

[1] *Chen, Y.Q., Vinagre, B.M. and Monje, C.A.* A Proposition for the Implementation of Non-integer PI Controllers. The Thematic Action 'Systems with Non-integer Derivations' LAP-ENSEIRB, Bordeaux, France,2003.

[2] *Leu, J.F., Tsay, S.Y. and Hwang, C*. Design of Optimal Fractional Order PID Controllers. Journal of the Chinese Institute of Chemical Engineers 33:2, 2002.

[3] *Podlubny, I.* Fractional Order Systems and $PI^{\lambda}D^{\mu}$ Controllers. IEEE Transactions on Automatic Control 44:1, 1999, pp. 208-214.

[4] *Bettoua, K. and Charef, A.* Control quality enhancement using fractional $PI^{\lambda}D^{\mu}$ controller. International Journal of Systems Science Vol. 40, No. 8, 2009, pp. 875-888.

[5] *Ziegler, J.G and Nichols, N.B*. Optimum Settings for Automatic Controllers. Transactions of the ASME 64, 1942, pp. 759-768.

[6] *A.N. Zavoryn, O.D. Yadryshnikov, V.A. Zhmud*. Improvement of quality characteristics of control systems with feedback when using PI2D2-controller. Collection of scientific works of NSTU. Novosibirsk. 2010. 4 (62). P.41 – 50 (In Russian, original name: *Заворин А.Н., Ядрышников О.Д., Жмудь В.А*. Усовершенствование качественных характеристик систем управления с обратной связью при использовании $ПИ^2Д^2$-регулятора. Сборник научных трудов НГТУ. Новосибирск. 2010. 4 (62). С.41 – 50).

[7] *Voevoda A.A., Zhmud V.A., Zavoryn A.N. and Yadryshnikov O.D.* Comparative analysis of optimization methods for regulators using VisSim and MATLAB software. Mechatronics, automation and control. № 9, 2012. p. 37–43 (In Russian, original name: *Воевода А.А., Жмудь В.А., А.Н. Заворин, О.Д. Ядрышников*. Сравнительный анализ методов оптимизации регуляторов с использованием программных средств VisSim и MATLAB. Мехатроника, автоматизация и управление. № 9, 2012. с. 37–43).

[8] *Poller B.V., Zhmud V.A., Novitsky S. P., Zavorin A. N and Yadryshnikov O.D.* Synthesis of a robust regulator by the method of double iterative parallel numerical optimization. Scientific bulletin of the NSTU. 2012. № 2. P. 196–200 (In Russian, original name: *Поллер Б. В., Жмудь В. А., Новицкий С. П.. Заворин А. Н, Ядрышников О.* Д. Синтез робастного регулятора методом двойной итеративной параллельной численной оптимизации. Научный вестник НГТУ. 2012. № 2 . С 196–200).

[9] *Zhmud V.A., Zavorin A.N., Polischuk A.V., and Yadryshnikov O.D.* The method of designing adaptive systems for the control of non-stationary objects with delay. Scientific bulletin of the NSTU. 2012. № 3. P. 172–177 (In Russian, original name: *Жмудь В. А., Заворин А.Н. , Полищук А.В., Ядрышников О. Д.* Метод проектирования адаптивных систем для управления нестационарными объектами с запаздыванием. Научный вестник НГТУ. 2012. №3. С. 172–177).

[10] *Chen, Y.Q., Vinagre, B.M. and Monje, C.A.* A Proposition for the Implementation of Non-integer PI Controllers. The Thematic Action 'Systems with Non-integer Derivations' LAP-ENSEIRB, Bordeaux, France, 2003.

[11] *Leu, J.F., Tsay, S.Y. and Hwang, C*. Design of Optimal Fractional Order PID Controllers. Journal of the Chinese Institute of Chemical Engineers 33:2, 2002.

[12] *Podlubny, I.* Fractional Order Systems and $PI^{\lambda}D^{\mu}$ Controllers. IEEE Transactions on Automatic Control 44:1, 1999, pp. 208-214.

[13] *Bettoua, K. and Charef, A.* Control quality enhancement using fractional $PI^{\lambda}D^{\mu}$ controller. International Journal of Systems Science Vol. 40, No. 8, 2009, pp. 875-888.

[14] *V. Zhmud, A. Zavorin*. Fractional PID-Controllers and Ways to Simplify Them with Increased Efficiency of Control. Automatics & Software Enginery. 2013. № 1 (3). p. 30–36. URL: http://www.jurnal.nips.ru/sites/default/files/ASE-1-2013-5.pdf

[15] *Zhe Yan, Kai Li*. Tuning and application of fractional order PID controllers. Proceedings of 2-nd International Conference on Measurements, Information and Control (ICMIC-2013). Harbin. China. P. 955–958.

[16] *Goldhordt V.G. and Ohm A.E.* Electronic block of frequency stabilization of lasers. PTE, 1980, P.190–193 (In Russian, original name: В.Г. Гольдордт, А.Э. Ом. Электронный блок стабилизации частоты лазеров. ПТЭ, 1980, с.190–193).

[17] *Dyakonov V.* VisSim+Mathcal+MATLAB. Visual mathematic modeling. Moscow. Solon-Press. 2004. (In Russian. Original name: Дьяконов В. VisSim+Mathcal+MATLAB. Визуальное математическое моделирование. Москва. Солон-Пресс. 2004). ISBN 5-98003-130-8

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